A simplified geotechnical risk-based approach for extraction level pillar design in Block/Panel caving mines

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Abstract

Definition of the extraction-level layout, including geometry and dimensions of the excavations, is an essential aspect of the design of any block/panel caving operation. The design needs to guarantee that the excavations remain stable and perform as intended during the life of mine, in other words, prior, during and after caving production takes place, a span of time that frequently involves decades. The layout of these excavations involves an intricate network of drifts, access tunnels, drawbells and other excavations. A complex 3D mechanical numerical model that incorporates the extraction level, the undercutting level and the broken material surfaces and which simulates the progression of the undercutting advance and resulting caving propagation was developed. The purpose of the model is to assess the layouts, in terms of stress concentrations and plastic damage in pillars at the extraction level, particularly in the vicinity of the caving front. A method for geotechnical risk-based design under high stress conditions which uses tridimensional numerical modelling and probabilistic methods of analysis to determine the probability of pillar failure is described in this paper. The methodology included three main tasks: (1) evaluation of the factor of safety (FOS) and probability of failure (POF) representative of the stability conditions of the extraction level layout; (2) evaluation of the risk associated with economic losses resulting from impacts on equipment and on production; and (3) generation of a simplified geotechnical risk map to compare several mining years. The results of these analyses enabled the identification of risk mitigation options for those situations where acceptability criteria are exceeded.

1 Introduction

In deep underground mines, where the production level is commonly located at great depth, a high concentration of stresses is expected on the infrastructure as well as on the mine operation. Because of these in-situ stress conditions, there is a need for a detailed stability analysis of the production level pillars and, in general, of the project layout. This will enable generation of a risk analysis to provide the basis for economic analysis and impact studies. The main objective of this article is to analyse the performance of production level pillars in the pre-mining abutment stress and relaxation areas, and with this, determine the potential existence of specific critical sectors. A more general objective is to observe how the pillars behave in regards to the mining progress as defined for the project by using numerical modelling. Based on these models and using the most relevant geotechnical parameters which control the stability of the area, a simplified geotechnical risk model was developed to evaluate the potential collapse area for a base case.

Collapses and rock bursts are the main geotechnical hazards that must be studied in more complete detail during projects' engineering stages and operations. There are other important geotechnical hazards, such as subsidence, water/mud rush and hangs up, but these are not part of this article.

A collapse is an instability of rather slow occurrence, characterized by a complete or almost complete convergence of a gallery or drift. This may occur because of an individual or fault failure of the crown pillar and/or the pillars of the extraction level in the affected area. The experience of the El Teniente mine, reviewed by Araneda & Sougarret (2008), indicates that collapses are mainly associated with the geometry and caving management, and the occurrence might be improved or worsened by the presence of unfavourably oriented geological faults. On the other hand, Karzulovic (2001) indicates that the main factors likely to trigger the a collapse would be: a) poor blasting operations that might leave remnant pillars in the undercutting level; b) isolated and irregular draw (Woodruff 1962), which allows the formation of stable arches in broken mineral deposits and the transmission of significant loading on to the extraction level pillars, and c) generation of point load due to coarse fragmentation in the undercut, which transfers significant local load weight to them (see Laubscher et al. 2017). This can be worsened by having a flat undercut height.

Worldwide collapse benchmarking (Figure 1), developed by Flores & Karzulovic (2002), indicates that:

i. The area affected by a single collapse varies from 140 to 17,500 m^2, with an average of 3,700 m^2.
ii. The main causes of collapse are: a) extraction rate/extraction management; b) the unfavourable presence of geological faults; and c) mine planning/mining sequence.
iii. The most frequent mitigation measures after a collapse are: a) to improve the extraction; b) more robust support designs; and c) improvements in the geological-geotechnical Information.

Finally, there are certain secondary factors that may worsen the occurrence of collapse in terms of process acceleration, thereby causing further damage or extending the affected area. These secondary factors would correspond to geometric singularities in the extraction level, pillars of insufficient size in the extraction level, insufficient fortification and degradation of the geotechnical quality of the rock mass produced by the opening of a drawbell/drawpoint and the abutment stress associated with the caving front.

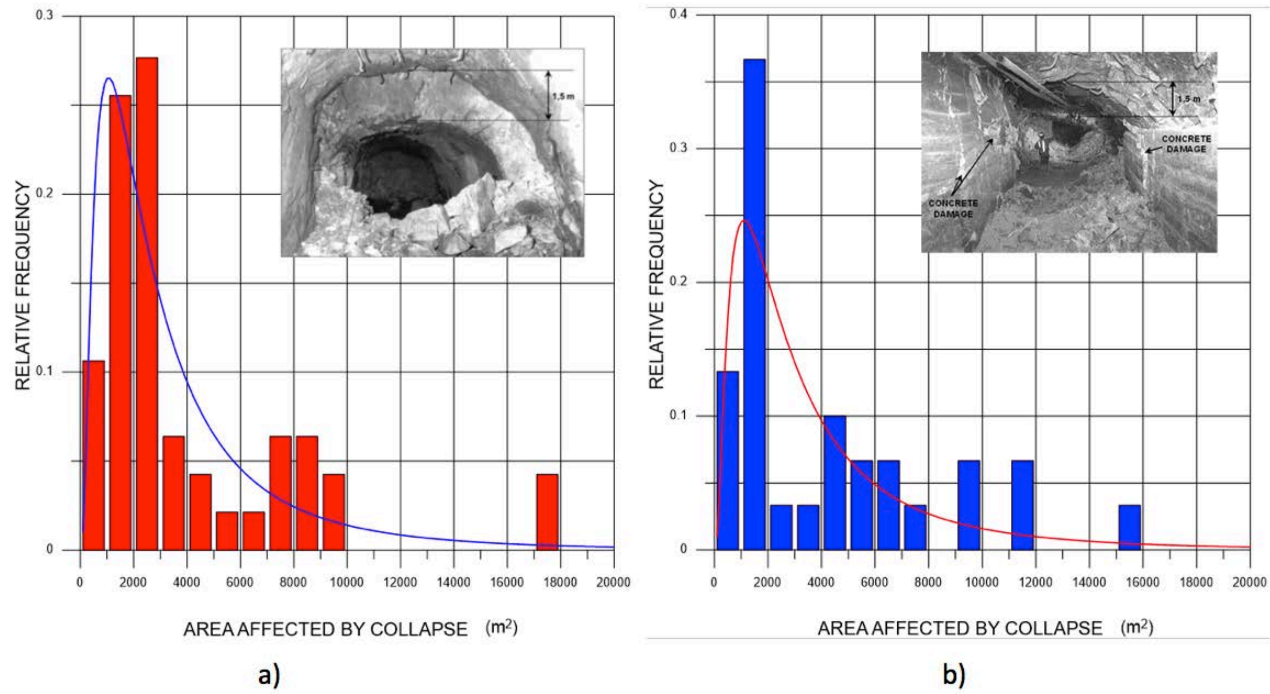

Figure 1 Histograms showing the relative frequency of the area affected by collapses in caving mines: a) worldwide benchmarking (modified from Flores & Karzulovic 2002) and b) Chilean caving mines (database from Villegas 2008)

The rock burst phenomenon is generated via a seismic condition induced by mining and produces a violent failure of a rock mass volume. It is a physical mechanism by which the rock is suddenly fractured due to the redistribution of stresses generated by the excavation (seismic event), and produces seismic waves and damage which impact the mine. There are various definitions of rock bursts based on energy rates, deformation energy density and resistance rate, among others. These are defined by several authors (Hawkes 1966; Peng 1986; Ortlepp 1997; Wang 2001; Stacey 2012), but the discussion and analysis of these methods and definitions are not within the scope of this article. Although the previously described geotechnical hazard will not be studied in detail in the present article (see Flores 2018; Laubscher et al. 2017), the key geotechnical factors and parameters that might trigger the above-mentioned phenomena will be analysed.

The conventional methodology for the pillar design is based on the calculation of stability indexes, such as the factor of safety (FoS) or the probability of failure (PoF). The indexes are compared with acceptability criteria to determine the appropriate performance or serviceability, which are then used in the extraction level design. An alternative design methodology is based on a quantitative assessment of the risks of pillar failure collapses, which can be used to apply the acceptability criteria for extraction level design. The risks are typically calculated in terms of economic and safety impacts, however, the paper discusses only the economic risks. The diagram in Figure 2 shows the risk assessment process for extraction level pillar design in terms of economic impact.

The main steps of the risk approach methodology described in this paper are:

1. Define a three-dimensional numerical model for pillar stability analysis to cover critical areas of the footprint during the life-of-mine (LOM). The objective is to have a proper representation of potential failure and collapse risks for mine plan evaluations.
2. To calculate reliability of the mine plan for each year represented in the collapse area, where probability density functions (PDF) for the collapse area are calculated by means of the Response Surface Method (RSM).

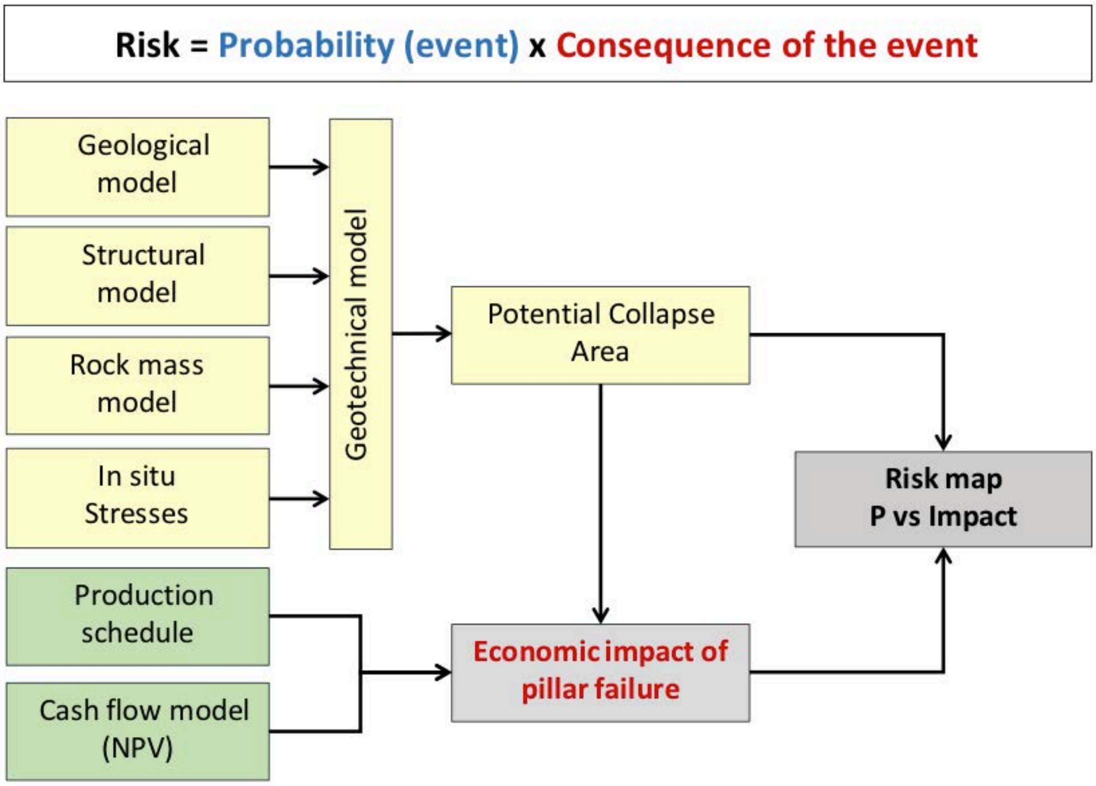

Figure 2 Simplified risk-based extraction level pillar design approach for economic impact (adapted from Contreras 2015)

3. Quantify the economic impacts of the collapse area, which are then included as a loss of annual profit or in the projected value as measured by the net present value (NPV).
4. Integrate the results of collapse area probability density functions and economic impacts for the construction of economic risk maps per year and for the LOM.
5. Compare the risk map to the project specific reference criteria to produce a risk assessment that is used in the decision-making process.

2 Geotechnical risk approach

2.1 General considerations

A risk model for the geotechnical extraction level design of a caving mine could be incorporated in the mining design and the mining macro sequence. The methodology of risk assessment provides more information about all the aspects of the exploitation level mining design and provides standardized values for many different exploitation strategies. With the risk assessment, the uncertainties of geology, rock resistance, stress condition, and the reliability of calculation methods are identified and quantified.

In contrast, with the application of the damage criteria, these aspects are hidden within the selection of what is considered an "acceptable" safety factor and the mine accepts an unknown range of risk. The risk assessment naturally complements the business decisions and changes the objective of a "stable pillars" design, to an evaluation of "acceptable" consequences (losses) if the pillars failure or collapse occurs. What is acceptable can be judged by the assessed economic benefit or with the cost of mitigation measures calculated in advance.

Generally, geotechnical engineers provide designs based only on the factor of safety and/or damage criteria. As a result, the inherent risk is adopted by the mine management, without having quantified or fully understood it.

The risk model provides the management with information on the range of real consequences of potential failures in mine pillars or the studied area. It also enables the management to have the opportunity and responsibility of determining the appropriate risk for their mining business (Steffen & Contreras 2007; Brown 2007; Wesseloo 2013; Joughin 2017). In addition, it provides a linear scale of risk judgement which contrasts the adoption of a single factor of safety with the methodology or damage criterion by geotechnical staff only. A higher factor of safety does not necessarily mean that there is a lower probability of failure. Finally, it allows the development of strategic management procedures for specific productive areas of the mine in question, and subsequently the evaluation of mitigation measures. Figure 3 shows the interaction between the factor of safety and the failure probability and risk when applied to underground mining.

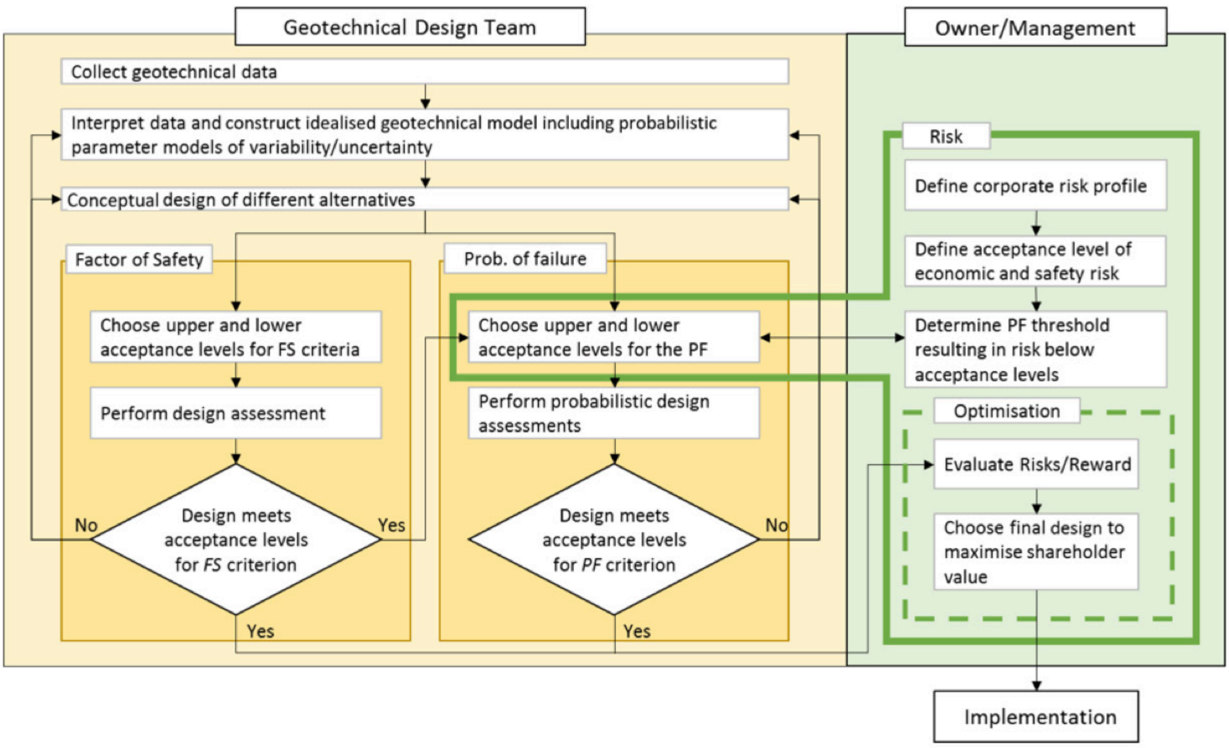

Figure 3 **Relationship between the criteria of acceptability for the factor of safety and the probability of failure and risk as design criteria (taken from Joughin 2017)**

2.2 Extraction level pillar stability

This section presents the analysis carried out on the production level pillars, mainly in order to study their performance and to determine the potential existence of critical sectors. In general, the numerical model allows a judgement on the stability condition of the pillars, so that a final probabilistic analysis, which allows for the geotechnical risk establishment associated with the project, can be carried out.

The objective is to study the effects of mining (excavation sequence, etc.) on the design proposed for the project. It specifically aims to determine the evolution of the principal stresses on the pillars as mining progresses. In order to do this, the stresses that could trigger stability problems were studied, and the areas with high stress concentration and/or with significant de-confinement were identified.

This method is widely known and used in stability analysis. It consists of evaluating the maximum stress resisted by the material with respect to the demanded stress, as presented in Ec.1. An advantage of this method is that it is widely used and easy to apply. It is currently implemented only for the Hoek-Brown criterion (Hoek & Brown 2019), but could be implemented with other criteria in the future.

$$FS = \frac{\sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a}{\sigma_1} = \frac{\sigma_{1\,max}}{\sigma_1} \quad (1)$$

In equation [1], σ_1 and σ_3 are the major and minor principal stresses, respectively; σ_{ci} is the unconfined compressive strength of the intact rock; and mb, a, and s are dimensionless parameters of the rock mass. The parameters mb, a, and s are defined according to the latest revisions of the Hoek-Brown failure criterion (Hoek et al. 2002; Hoek & Brown 2019).

The performance of the pillars basically depends on the ratio between the resistance and the forces acting on them. This means:

- The pillar strength depends on the geological-geotechnical conditions of the rock mass and the pillar geometry, as well as on the accumulated damage to the pillar, which itself depends on the blasting on the UCL level, and the time the pillar is exposed to stress concentrations (abutment stress zones).
- The acting stresses on the pillar depend on the in-situ tensional states and the stresses induced by mining.
- The in-situ tensional states depend on the depth, geological history of the deposit, the surface topography and the ratio of stresses (the latter being influenced by the first two factors).
- The stresses induced by mining depend on the characteristics of the exploitation method, which for this case, are determined mainly by the geometry of the extraction and undermining fronts, the type of undermine, the height of the block and the form of caving propagation.

Several formulas have been proposed in the technical literature in order to evaluate the resistance of a rock pillar, R_P (e.g., see Brady & Brown 1993). However, in general, with respect to all of them, the following can be said:

- The strength of the pillar depends on the type of rock, the volume and the geometry of the pillar.
- The effect of the type of rock is determined by its "pattern strength", R_0, representing the strength of the rock mass that constitutes the pillar body. It is usually defined in terms of a uniaxial compression strength.
- The effect of the volume of the pillar, V_P, is typically included as a function, calibrated based on a specific database.
- The effect of the geometry of the pillar is generally included as a function of its width, W_P, and its height, H_P, or as a function of its slenderness, or ratio height/width, E_P.

Then in general, the pillar strength is given by the following formula:

$$RP = R0 \times V_p^a \times \frac{W_p^b}{H_p^c} \tag{2}$$

or

$$RP = R0 \times V_p^m \times E_p^n \tag{3}$$

where a, b and c (or *m* and *n*) are empirical coefficients based on a specific database (usually obtained from productive sectors or different mine sites).

On the other hand, all pillars can be considered to be approximately the same height and the same width in the case of block/panel cave mining, (independently of the values of b and c) and are based on the previous equation:

$$\frac{W_p^b}{H_p^c} = K = constant \tag{4}$$

and for practical purposes, the pillar strength can be assumed to be proportional to the overall strength in uniaxial compression of a volume of rock mass:

$$RP = \left(R0 \times V_p^a\right) \times K \tag{5}$$

Thus, in order to determine the pillar stability, it should be taken into account that the pillar is a volume of rock mass with a certain strength and will be affected by a non-uniform distribution of stresses, deformations, etc. The above can be obtained from the results of the numerical models and can be compared to the acceptability criteria (Figure 4). The details of the methodology, assumptions and applicability are presented in Hormazabal et al. (2018).

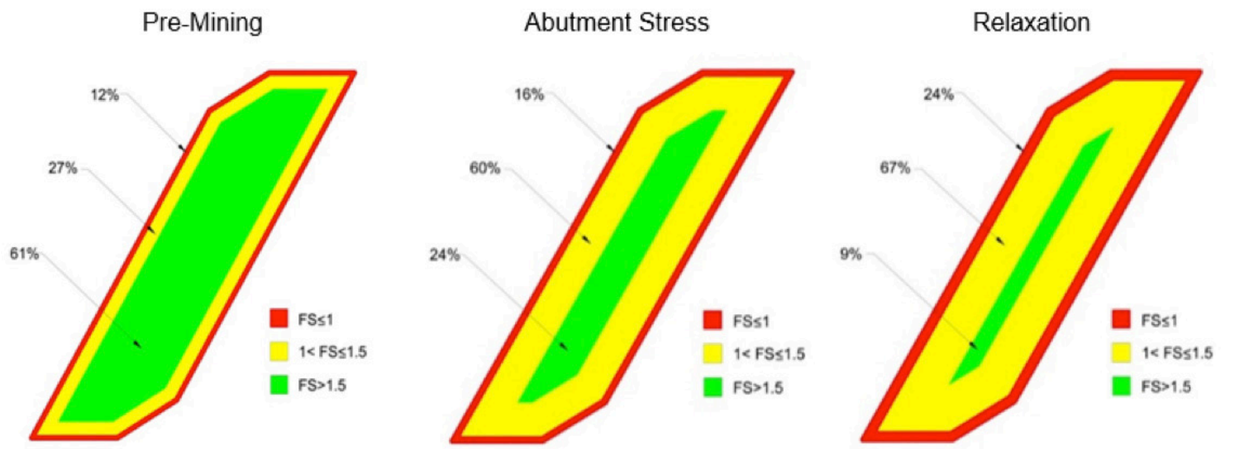

Figure 4 Illustrative distribution scheme of factors of safety within a pillar affected by the main mining stages in a caving operation (taken from Hormazabal et al. 2018)

2.3 Acceptability criteria

Any assessment of the stability condition of a pillar must consider the potential modes of failure, its strength, and the magnitude of the demand (stresses) to which it is subjected. When referring to the pillar condition, an index of the pillar stability is generally used, the factor of safety being the most common (the ratio between capacity and demand). However, the safety margin (difference between the capacity and demand), the probability of failure, and/or other variables could also be used. The pillar condition is commonly assessed quantitatively by comparing this index with some acceptability criteria.

For this case, the factor of safety has been used in order to determine the stability of the pillars because of the simplicity of application to the numerical model and the subsequent use in the simplified geotechnical risk model implemented here. There are other approaches that could be considered, such as total strain based on visual observations and geotechnical monitoring (see Cavieres et al. 2003) and deviatoric stress based damage initiation and propagation (see Martin et al. 1999; Hajiabdolmajid & Kaiser 2003; Cai et al. 2004; Diederichs 2007; Villegas et al. 2009; Kaiser 2010; Estherhuizen et al. 2018).

The acceptability criteria used here stipulates that the condition of a pillar without drawpoints is acceptable if at least 70% of the volume of the lower pillar has an FS ≥ 1. It is unacceptable, and the pillar loses its serviceability, if more than 30% of the lower pillar volume has an FS ≤ 1. For pillars with drawpoints, the condition is unacceptable or serviceability is lost if 15% or more of the volume of the lower pillar has an FS ≤ 1. It could be argued that the 70% value would be too high (conservative) for operational sectors where ground conditions, faults, in-situ stresses are known (Balboa et al. 2018). Nevertheless, when considering a greenfield project, this value seems reasonable according to the experience of the authors of this work. Similar threshold values have been applied to pillars of the operational sectors of the El Teniente mine (Vázquez et al. 2008).

As part of this study, complex three-dimensional continuum models have been developed and applied to evaluate the influence of the above mentioned variables in the mechanical response of the underground operations. In particular, they have been applied to stress concentration that developed in critical areas such as pillars in ore pass excavations. Table 1 lists geometrical and geotechnical parameters used in the tridimensional model.

Figure 5 shows an isometric view of the Flac3D (Itasca 2015) model output. To facilitate the interpretation of the results, each pillar in the numerical model is considered as an independent volume with a unique ID. Also, each pillar is subdivided into an “upper pillar” (UP) and a “lower pillar” (LP), with the limiting horizontal boundary located at the top-height of the production drifts. Taking that into account, the LP (between extraction level floor and the aforementioned horizontal boundary) shows a higher excavation rate in the mining layout, and this LP is considered to be the region where major damage takes place. For this reason, this LP was the focus of the analysis.

Table 1 Geometrical parameters used in the three-dimensional model

Parameters			Values
UCL	Drifts	Spacing (m)	32
		Height (m)	4.0
		Width (m)	4.0
	Undercut height (m)		10
	Undercutting Variant		Conventional (post)
	Undercutting Type		Flat Undercut
EXL	Drifts	Crown-Pillar (m)	18
		LHD Layout Type	El Teniente
		Spacing (m)	32
		Height and Width (m)	4.6
Drawpoints		Spacing (m)	17

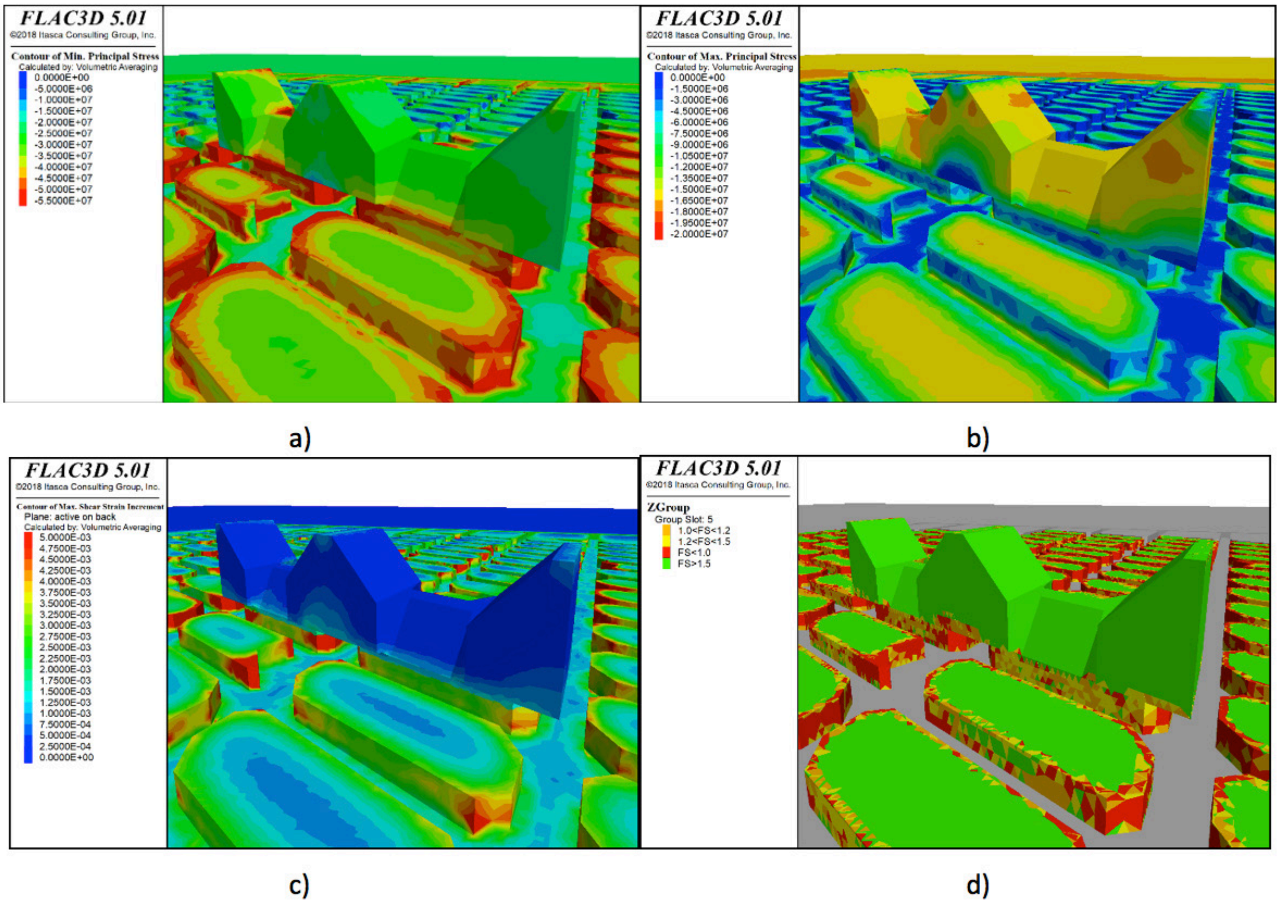

Figure 5 **Isometric view of a three-dimensional numerical model used to analyse the stress concentration in pillars of a caving operation: a) major principal stress b) minor principal stress c) maximum shear strain increments d) factor of safety**

Figure 6 shows the evolution of the number of pillars in stable and unstable conditions for the mining years considered in this analysis. A FISH function has been written into the numerical model code which enables separation by colour. In red are the lower pillars that do not meet the acceptability criteria proposed here (FS ≤ 1 in 30% of the volume of the lower pillar), and in green the lower pillars that present a condition of FS > 1 in more than 70% of the pillar volume. Figure 7 shows the distribution of pillar failure per year. During the first 2 years there were no pillar failures, partly because only the developments and construction of the drifts and galleries were being carried out at that time.

2.4 Probability of collapsed area

For this project, a random component has been added in order to incorporate the natural variability and uncertainty of the parameters. This allows a statistical measurement of the performance of the model using the probability density function (PDF) for the collapsed area percentage (the percentage of collapsed pillars), which in turn represents the reliability of the pillar layout.

This methodology is similar to the methodologies used for obtaining the factor of safety (used in the slope probability of failure calculation), but in this case the random variable is not the FoS but the percentage of the collapsed area.

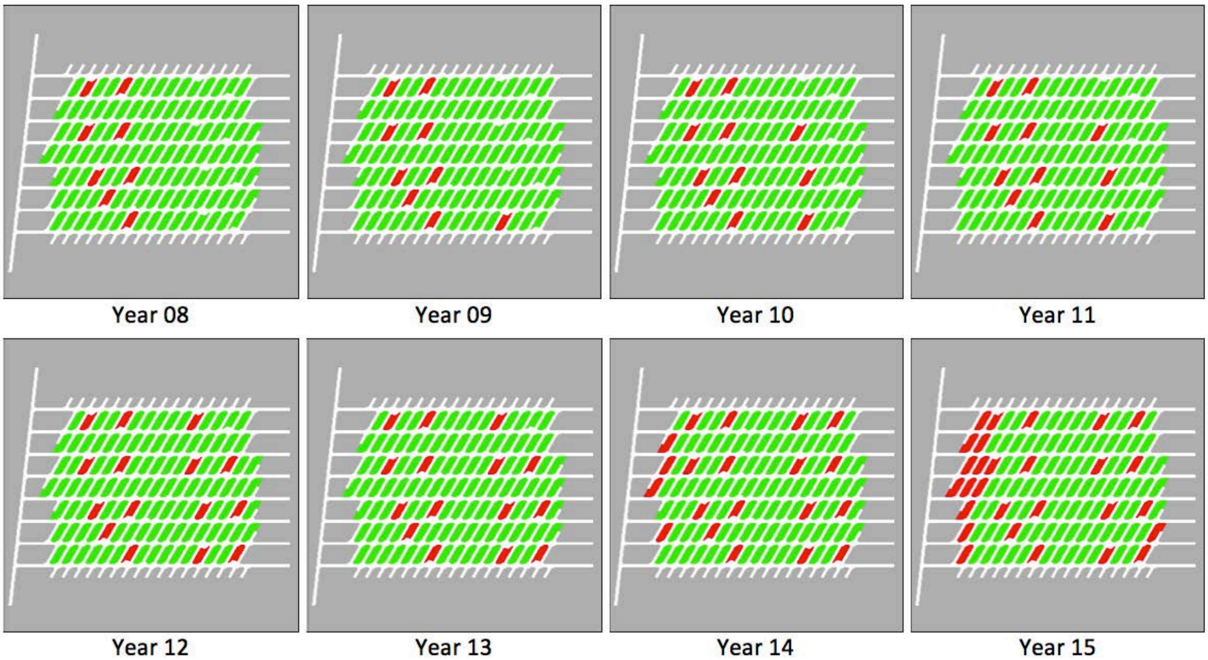

Figure 6 **Production level plan view. Distribution of failed pillars according to the acceptability criteria**

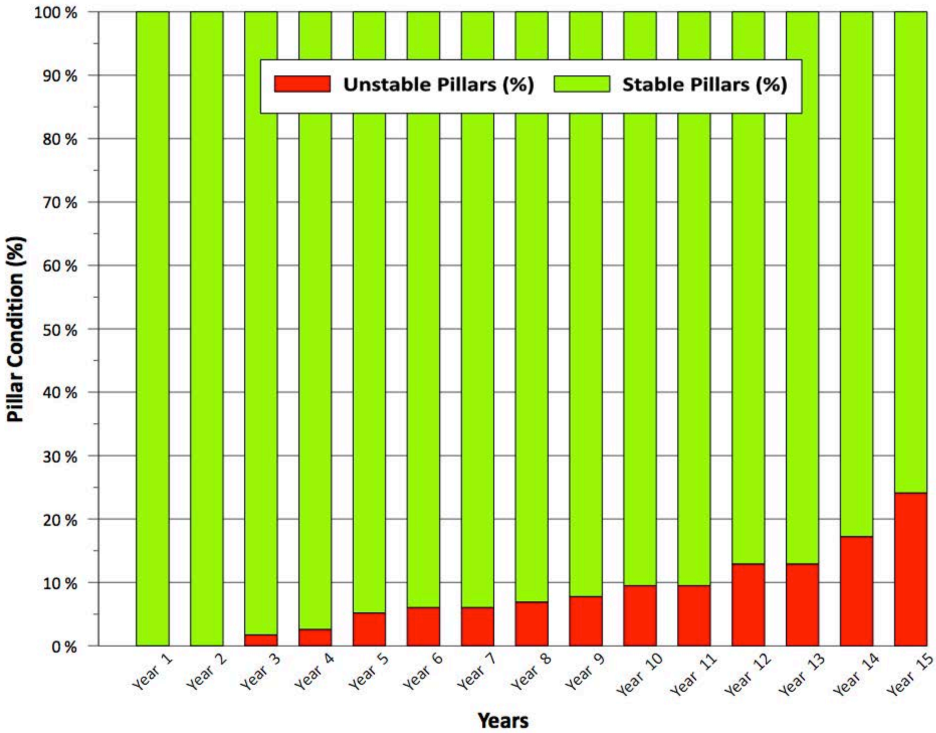

Figure 7 **Evolution of pillar condition per year according to the acceptability criteria adopted**

There are several methods to calculate the PDF of the collapsed area, which can be roughly classified in:

- Methods of moments estimations, e.g., First-Order Second-Moment Method (FOSM), Point Estimate Method (PEM), etc.
- Methods based on sampling, e.g., Monte Carlo Method (MCS), simulation of subsets (SS), Importance Sampling (IS), etc.

The methods of moments estimation in general assumes a family of probability distribution (e.g. normal) for the target function (percentage of the collapsed area), and approximates its first and second moments (average and variance) through analytical expressions or specific estimations of the function.

Among these methods, generally called FOSM, are the Taylor series (Ang &Tang 1975) and the PEM method (Rosenblueth 1975). The Taylor series is an analytical expression of the target function and its derivatives. The PEM method uses estimators that characterize the distribution of the inputs, such as moments (e.g., average and standard deviation), and subsequently calculates the specific estimators of the target function (PDF of loss area). This method produces specific estimators, but not the full distribution (unless assuming a priori the distribution of the output).

Because the target function for this analysis corresponds to the three-dimensional model, it is not possible to use the Taylor series method. Due to the limited number of runs, the PEM method is useful for this case. For the PEM method, two runs are necessary in its simplest version, with n being the number of variables. It has the same computational cost as a full factorial design (Montgomery 2001).

Finally, the methods based on sampling or direct methods are non-deterministic methods, where the PDF of the random function is obtained from results of multiple experiments. The distributions of inputs are sampled in each of them and responses generated that would it be difficult or impossible to obtain analytically. The direct use of these types of methods with computational models is impractical since a large number of runs, each of which may consume many days or even weeks, is required.

Because of this, it is common to use surrogate models that approximate the response of the real model through an adjustment of real results to specific functions. Among these methods are: support vector machines, neuronal networks and the response surface method (RSM). This latter method, developed within the discipline of experimental design, has been previously used in surface mining (Steffen et al. 2008; Contreras et al. 2019), as well as underground mining (Langford & Diederichs 2015; Lu & Low 2011; Joughin 2017).

2.5 Description of the methodology

To calculate the probability density function of a collapsed area (percentage) the response surface method is used to approximate the solution of the numerical model and the Monte Carlo method to propagate the variability of the inputs in the response. First, the inputs and outputs to be used are defined. For the output, the variable to be represented by the response surface is the percentage of unstable pillars according to the adopted acceptability criteria. As inputs, the variables with the greatest impact on the output, but which also have great variability, should be chosen. The methodology for calculating the PDF is shown in Figure 8:

- Step (1) is the experimental design, that is, the selection of the variables to be considered in the analysis (a and b in the example), and their low and high values, plus the definition of the combinations of parameters that will be modelled. An experimental star design will be used, which has already been formerly used in slope applications (Steffen et al. 2008; Contreras et al. 2019). Although this method confuses the cross and side effects, it has the advantage of needing a low number of runs ($2n + 1$, with n being the number of parameters), even when compared to a 2-level factorial design. Another advantage is that it is easy to add new variables, as opposed to other types of designs. In addition, the definition of cross effects can be improved by adding a factorial design, thus generating a central composite design (CCD).
- Step (2) consists of running the three-dimensional numerical models using the parameters defined in step (1). From this the percentage of failed pillars will be obtained and this will be used to approximate the response surface.
- In step (3) the response surface is constructed based on the results of step (2). In order to do this, first the value of the variable of interest at the central point of the design is calculated, y^{base}. Then, using the experiment results for each parameter at its low and high value, β^k coefficients are calculated, which act as factors of value scaling at the central point. With these scaling factors,

linear functions are adjusted by sections, allowing the representation of the target variable derivatives around the central point. This procedure assumes that there is no correlation between the effects of the independent variables.

- In step (4) the probability distribution functions (PDFs) are defined for the uncertain parameters. Using these, a Monte Carlo simulation is performed in order to obtain sets of parameters $\{a_i, b_i,...,z_i\}$, where a_i, b_i... are values of each parameter in set i. This permits normal or log-normal distributions to be taken into account without the risk of generating extreme values outside the established range for each variable.
- Finally, the response surface is evaluated with each of the parameter sets, and with these results, in step (5) the probability density function of the percentage of pillars in a state of failure.

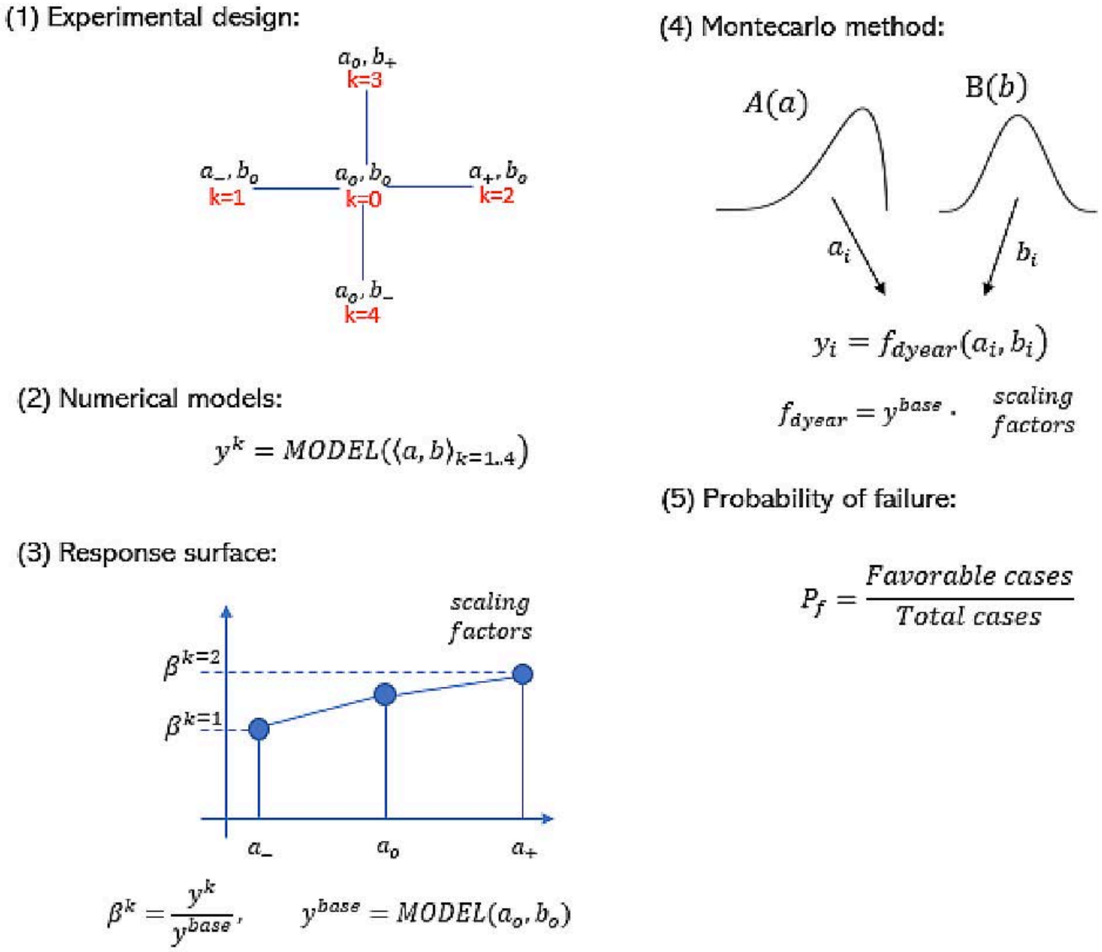

Figure 8 **Graphic representation of the methodology for calculating the probability density function**

2.6 Application of the methodology

The probability of having a collapsed area was determined according to the acceptability criteria previously described for each year of the mine plan. The probability of collapse was calculated by using key geotechnical parameters, such as: the value of rock mass quality in terms of the Geological Strength Index, or GSI, (Hoek & Brown 2019), the east-west stress ratio (k_{ew}), the north-south stress ratio (k_{ns}) and the uniaxial compression strength (UCS). The probability of lost area in the simplified geotechnical risk model was evaluated by a Monte Carlo analysis, using the response surface method (Morgan & Herion 1990). With this methodology the factor of safety (FoS) was calculated for all the analysed cases. The best estimated values of all the variables, as well as of the extreme cases for each uncertain variable, were used as input data.

In agreement with the methodology of section 2.5, a total of nine cases were considered, corresponding to the variation of four parameters and one base case. The low (-), average (0) and high (+) values of each variable for the conventional Hoek-Brown case are presented in Table 2. The following probability distributions were used for the problem inputs:

- For the stress ratios, symmetric triangular distributions with their boundaries given by the low (-) and high (+) values, and the most likely value equal to the average value (0). For the stress ratios K_{NS} and K_{EW}, a variation of ± 0.15 was used.
- For the GSI, a triangular distribution with the probable value equal to the GSI value, and its boundaries given by GSI ± 10.
- For the uniaxial compressive strength σ_{ci}, a log-normal distribution was used, with standard deviation of 20% of the average (arithmetic).
- The Crystal Ball software was used (Oracle 2017) to perform the Monte Carlo type simulations.
- The random variable studied corresponds to the percentage of the lost area (PP) and its probability distribution is the result of the Monte Carlo simulation. Collapsed area = PP% x A_T (where A_T is the tributary area per pillar = 544 m^2).

Table 2 Cases modelled for variability in properties with Hoek Brown model

Case	GSI [mb]	KNS	KEW	σ_{ci} (MPa)
Base	65 [5.73]	0.8	1.2	100
1	**55 [4.01]**	0.8	1.2	100
2	**75 [8.19]**	0.8	1.2	100
3	65 [5.73]	**0.65**	1.2	100
4	65 [5.73]	**0.95**	1.2	100
5	65 [5.73]	0.8	**1.05**	100
6	65 [5.73]	0.8	**1.35**	100
7	65 [5.73]	0.8	1.2	**80**
8	65 [5.73]	0.8	1.2	**120**

3 Economic impact and risk map application

The measure of value of a mining plan is usually represented by the NPV. The NPV is normally the result of a mine plan optimization process carried out with specialized software with the goal of maximizing the economic benefits. In general, the economic impact of a pillar failure or an area collapse event is a result of the ore feed schedule disruption during the time required to rehabilitate the site and from the additional costs incurred by these activities. The economic impact of the collapsed areas is calculated as the difference between the NPV of reference (i.e., the mine plan without collapsed areas) and the NPV with the effects of the collapsed area.

Production change and disruption may be caused by factors such as an interruption in access to the mining areas, ore coverage, and grade variations resulting from the use of alternative ore sources to mitigate the effects of the collapse. Additional costs are a result of repairing drawpoints and ore passes, and the re-scheduling of equipment during the implementation period.

The first step for estimating the economic impact of pillar failure consists of quantifying the collapse area, which is directly related to the magnitude of the impact on production and costs. The collapse area estimation is based on the tributary pillar area times the number of failed pillars. The estimation uses the results of a tri-dimensional geomechanical analysis and a simple geometrical criterion applied to the collapse area (collapse area x column height), which is sufficient to estimate the relative effect on the production plan.

The estimation of impact on production is a matter for expert judgement, but the impact factors need to be supported by reference calculations in order to capture the relative effects of the collapse events. Some events would not have an impact on productive sectors, but in these cases, the impact would be reflected in the additional costs to restore the drawpoints, galleries and/or production drifts.

The integration of the results of collapsed area PDF for each year of the mine plan and economic impact described in the previous sections are used to construct risk maps per year and for the LOM. The risk map describes the relationship between the probability of having a given economic impact and the magnitude of that impact. This concept was originally developed for the open pit cases (Contreras 2015).

The risk map results for the mine plan showing the extraction level collapse areas are shown in Figure 9a. These results are based on the PF and impacts on NPV described in previous sections. These graphs show the relationship between the probability of exceeding the impact on NPV and the magnitude of the impact. The envelopes correspond to the cumulative probability distribution curves of the loss areas and define the economic risk profile in terms of pillar failures (areas) of the respective year.

The complete economic risk map is shown in Figure 9b. This graph groups the risk envelopes for each year of the mine plan, as well as the envelope indicating the cumulative risk for the LOM. The procedure to define the LOM risk envelope is based on adding the probabilities from different years for specific impact values, using the concept of system reliability (Harr, 1996). The probability of an annual economic impact during the LOM (PLOM) in Figure 9b is calculated using Equation 6:

$$P_{LOM} = 1 - (1-P_{2019}) \times (1-P_{2020}) \times (1-P_{2021}) \times (1-P_{2022}) \times (1-P_{2023}) \times (1-P_{2024}) \times (1-P_{2025}) \times (1-P_{2026}) \times (1-P_{2027}) \times (1-P_{2028}) \qquad (6)$$

where:

P_{year} = Annual probability of economic impact in the indicated year.

The interpretation of the simplified economic risk envelope for the LOM in terms of impact on NPV is shown in Figure 9a, where the pointed value corresponds to the 50% probability of having an impact of at least 8 % on the annual profit during the ten years of the mining plan. This graph is useful in identifying the most relevant extraction level years in terms of potential economic impacts and to assess mitigation strategies to reduce risks.

4 Conclusion

This risk evaluation process is based on the construction of simplified risk maps for the economic impact of collapse areas. It takes into account the probability of pillar failure obtained through geotechnical analysis and numerical modelling, and the economic impact of those collapses on the cash flow analysis of the mine plan. Both types of analyses are normally based on limited data and require subjective inputs from engineering judgement and expert opinion. For this reason, it is important to carry out a tuning analysis of the models used to validate their results. Numerical model tuning for the calculation of the pillar failure probability is based on observed conditions of the operating sectors and ground control inspections. The validation of the economic impacts estimated with cash flow analysis are based on historical cases of collapses around the world, with an emphasis on Chile, where their detailed mining scheduling is done in real collapse situations.

Based on the experience of block caving in mines in Chile, and the results obtained from the numerical analyses presented in this study, several macro-sequences can be analyzed. In addition, mining strategies, fall back alternatives and remedial alternatives can be identified by using this simplified risk map approach in order to transfer, avoid, reduce or accept the risk and economic impact.

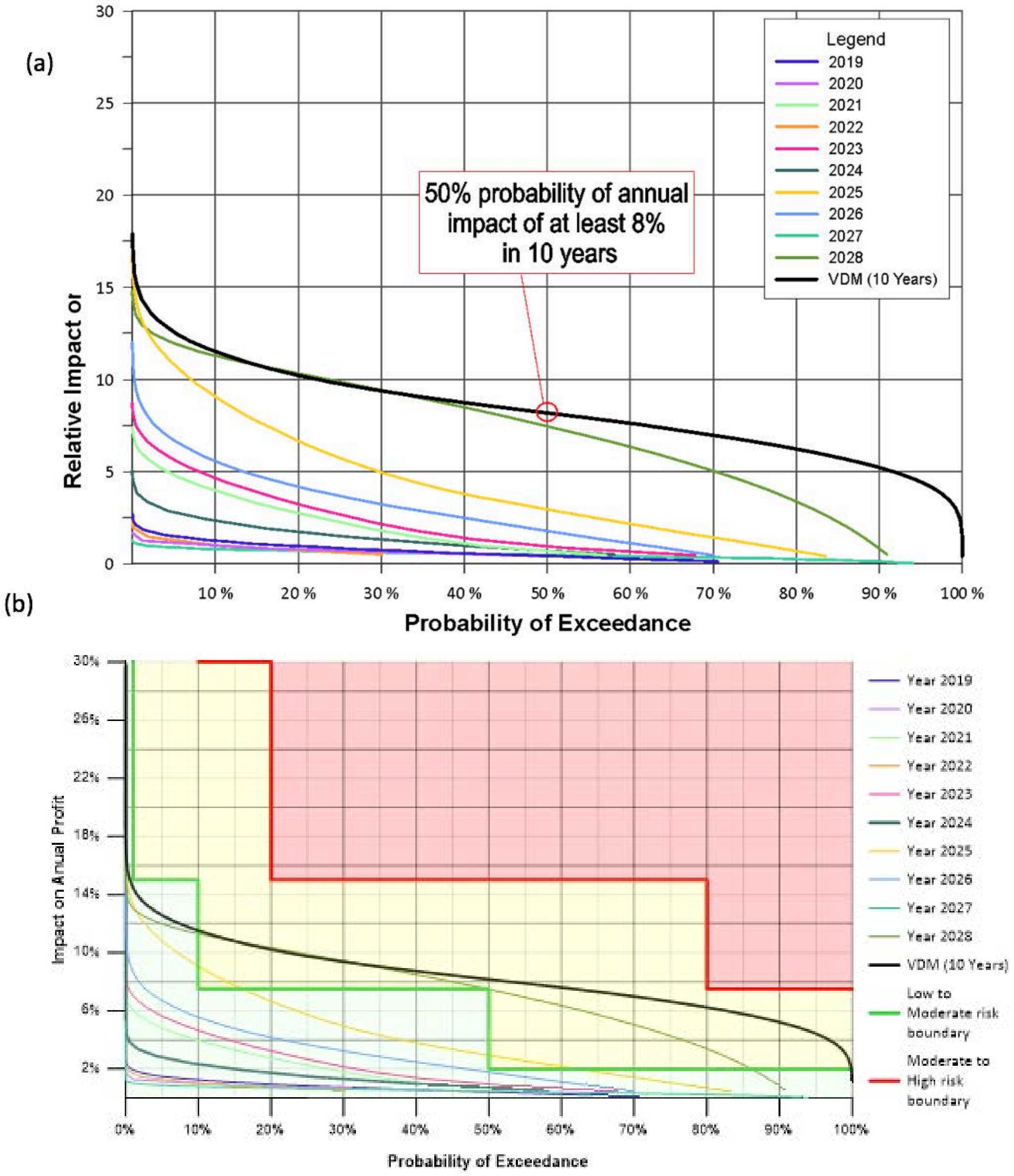

Figure 9 Simplified economic risk map for impact on NPV: (a) Relative impact on annual profit (b) reference criteria compared with the risk maps for impacts on annual profits for the LOM

One of the big advantages of using the simplified risk map approach is that different types of numerical models can be used, different acceptability criteria can be adopted, different failure mechanics and constitutive models can be used, based on the rock mass behavior. Also, the risk acceptability matrix can be modified according to the mining company´s vision and risk management approach.

The risk methodology for economic impact of collapse areas was used to evaluate a typical block caving operation. The risk map application could be improved by incorporating geological faults, lithological contacts and multiple possible occurrences for a given year. This would permit an analysis of the macro sequence and determine the best mining alternative.

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