# Empirical mill throughput modelling and linear programming for blend optimisation at the Phu Kham copper-gold operation, Laos

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## ABSTRACT

PanAust's Phu Kham copper-gold operation (Phu Kham) in Laos is currently limited by throughput in the Semi Autogenous Grinding (SAG) mill. The ability to accurately predict SAG mill throughput (throughput) is therefore critical to accurately forecast copper and gold production. During the fourth quarter of 2016 the proportion of hard unweathered rocks in the mill feed increased, resulting in a period of lower than expected throughput. These low throughput rates prompted a study with the aim of identifying a practical solution for throughput predictions. The study identified an empirical modelling approach based on actual SAG mill throughput rates. The empirical model was then optimised by linear programming to identify the ideal feed blend of unweathered rocks to achieve a maximum SAG mill throughput.

Rock strength was the initial focus of the throughput study at Phu Kham because previous studies had identified a strong link between rock strength and throughput. It was determined that there was insufficient valid data to adequately model but sufficient data to qualitatively characterise rock strength. The data identified that the weathering and lithology had a controlling influence on the rock strength. Complicating throughput predictions were operational improvements (changes to blasting and mill settings) implemented in response to the lower than expected throughput. This created uncertainty with the throughput equations. A self-learning empirical modelling method was developed to predict the throughput by incorporating fundamental rock properties with operational practices and improvements. The method is based on the proportion of mill feed with similar weathering and lithology (a proxy for comminution performance) and the actual mill throughput using the SAG mill as the analytical instrument. The modelling method simultaneously incorporated the sum and interaction of unmodelled influences including blasting, crushing and mill settings. The empirical modelling method worked at Phu Kham because the lithology and weathering has an intuitive and observable controlling influence on SAG mill throughput. The modelling method is described along with a worked example of feed blend optimisation using linear programming.

## INTRODUCTION

It has long been recognised that processing bottlenecks can significantly affect a mines performance. At PanAust's Phu Kham copper-gold operation in Laos the processing bottleneck is, for most of the part, the SAG mill. The ability to reasonably predict the comminution performance of the ore is important for optimising the mining operation but critical when the operation is throughput constrained. Failing to achieve a desired outcome in terms of copper and gold produced can have a major financial impact on a business. An inaccurate throughput model may lead to inaccurate mine plans which can significantly impact on cash flow. Long-term planning for ore selection, mining and logistics fleet, staff, maintenance, consumables and many other aspects rely on accurate life-of-mine (LOM) models.

The SAG mill throughput is dependent on many factors including, but not limited to, the feed lithology, weathering, natural fractures, alteration, intact rock strength, rock fabric, blasting, crushing and mill maintenance and operation. Accurate modelling and predicting the likely throughput of the mineralised rocks through to the LOM is the goal of throughput modelling. It is clearly possible to do this, as demonstrated at many mines around the world.

### Brief description of the Phu Kham copper-gold operation

The Phu Kham copper-gold operation comprises a large-scale, open cut mine located approximately 100 km north-east

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of the Laos capital Vientiane. Phu Kham is Phu Bia Mining's flagship Operation (Figure 1). Phu Bia Mining Limited (PBM) is the Lao-registered subsidiary PanAust Limited. PBM is 90 per cent owned by PanAust with the Government of Laos holding the remaining 10 per cent.

Production commenced at Phu Kham in 2008. PBM's other mine in Laos, the Ban Houayxai Gold-Silver Operation, is located 25 kilometres from west of Phu Kham. Production commenced at Ban Houayxai in 2012.

The Phu Kham deposit has complex mineralogy consisting of copper-gold stock work and skarn mineralisation (Tate, 2005). Weathering has created a soft oxidised zone, overlying transition zones with supergene enriched chalcocite-dominant secondary copper mineralisation and clay-rich gangue. The rock mass strength and degree of weathering varies considerably across the deposit with rocks with higher intact strength found in the deeper levels. This variability causes a wide-range of plant throughput and metallurgical performances.

The Phu Kham Operation comprises an open cut mine feeding ore to a process plant with recovery of copper and precious metals into a saleable concentrate using conventional flotation technology (Bennett *et al*, 2014). Expansion of the Phu Kham process plant in 2012 increased the maximum design capacity from 14 Mt/a to 18 Mt/a through additional grinding and rougher flotation capacity. Further debottlenecking in 2013 saw throughput achievements of 20 Mt/a when feeding softer ores.

Crushing is performed in a single stage with a gyratory crusher. The grinding circuit consists of a SAG mill and two parallel ball mills each in closed circuit with hydrocyclones. The flotation circuit comprises roughers, regrind and several cleaning stages. The concentrate contains between 23 and 25 per cent copper, up to 9 g/t gold and 60 g/t silver.

The last publicly reported Mineral Resource was on 31 December 2015 with 164 Mt at 0.49 per cent copper, 0.21 g/t gold and 2.1 g/t silver. The Mineral Resource and Ore Reserve models have reconciled well in 2016 against milled tonnes (within one per cent) and copper grades (within three per cent, relative).

## Brief description of regional and deposit scale geology

The Phu Kham deposit is located at the northern end of the north-west-south-east trending Truong Son fold belt (Tate, 2005 and Kamvong *et al*, 2014). The fold belt is 50 to 100 km wide and greater than 1000 km long, extending north-west across Laos and north-west Vietnam. The Truong Son fold belt has a complex tectonic history and consists of Palaeozoic to early Mesozoic volcano-plutonic suites and hosts several deposits including Chatree, Phu Kham, Ban Houayxai and Sepon. The Loei fold belt intersects the Truong Son fold belt near Phu Kham.

Mineralisation at Phu Kham occurs in skarn and stockwork styles that represent the distal expression of a conventional porphyry copper-gold mineralisation system (Tate, 2005). The deposit occurs in a sequence of deformed and hydrothermally altered tuffs, volcaniclastics and carbonate sediments of late Carboniferous age (304 million years ago) with a crowded feldspar porphyry diorite having a U-Pb zircon age of 291 million years (Tate, 2005). The host sequence forms the upper plate of a thrust fault. Beneath the thrust is a basement of redbed siltstones and arkosic conglomerates intruded by Silurian granite (Figure 2).

Almost all of the host sequence is affected by an envelope of moderate to strong phyllic alteration. Intermittent propylitic alteration assemblages occur just outside the envelope. Smaller zones of potassic and skarn-style alteration occur within the envelope. Strong silicification occurs within acid intrusives and as halos around some structures. A low angle fault between the host sequence and the basement dips north-east at ~20°. The structure varies from a 30 cm wide shear to a 20 m thick swarm of unmineralised massive quartz veins. The low angle fault is cut by a series of north to north-west vertical faults.

The majority of mineralisation occurs as a stockwork in altered volcaniclastics and diorite with the remainder as distinct skarns and veins. Stockwork mineralisation occurs in the altered tuffs and volcaniclastics of the host sequence and in the crowded feldspar porphyry diorite. Grades in the stockwork vary between 0.1 and 1.0 per cent copper and 0.1 to 0.5 g/t gold.

**FIG 1** – Phu Kham copper-gold operation open cut, March 2017.

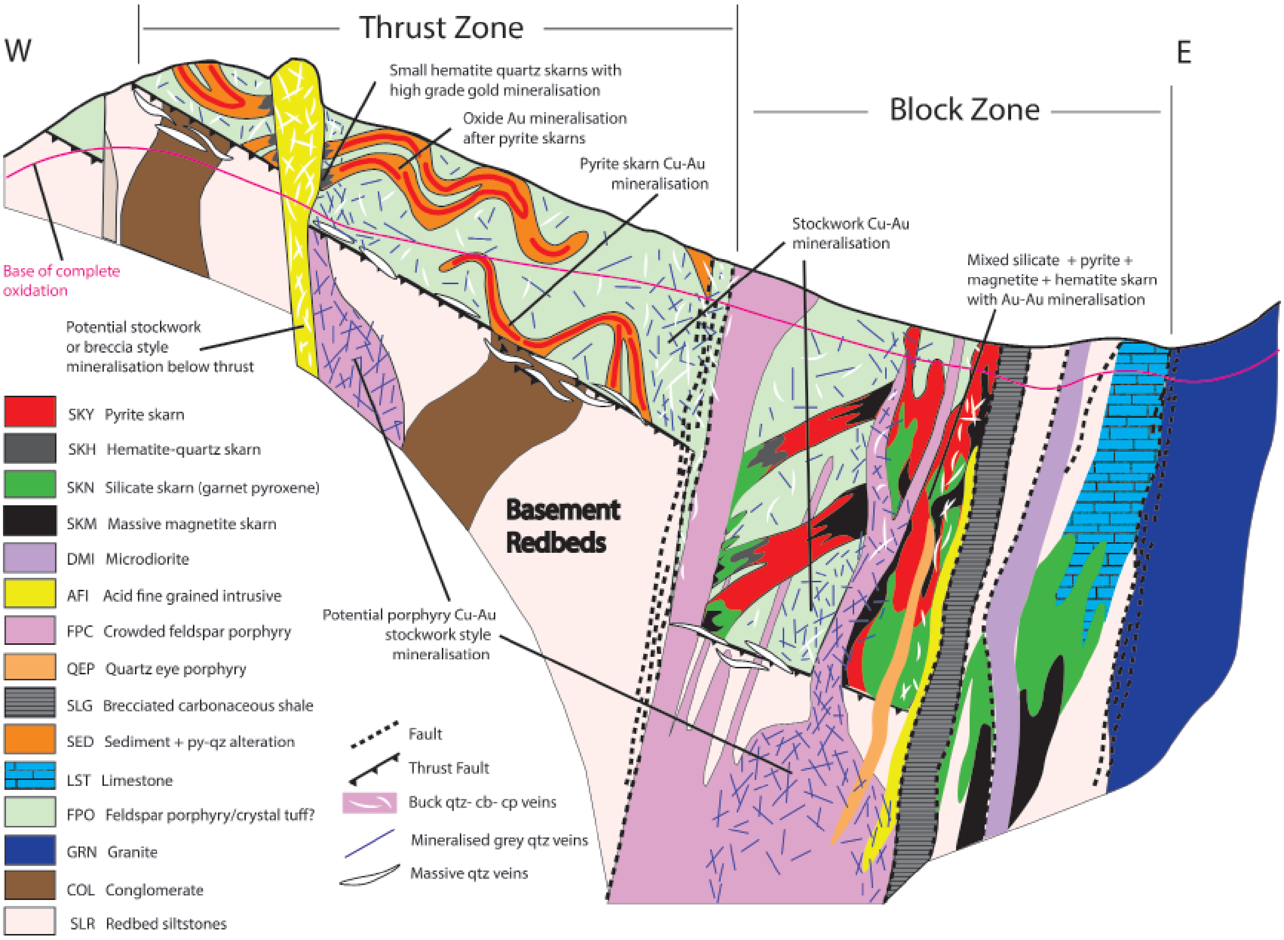

**FIG 2** – Geological concepts cross-section of Phu Kham (modified from Tate, 2005).

In 2016, higher proportions of hypogene magnetite skarns were being fed into the SAG mill. The magnetite skarns contained copper grades exceeding 1.5 per cent copper and dry bulk densities exceeding 4 t/m$^3$.

## Existing SAG mill throughput model

The existing throughput model was based on a measure of intact rock strength, the point load index (PLI) Is50 value. The PLI was identified by PanAust metallurgists to have the best predictive properties for determining the drop weight index (DWi). The DWi is a strong predictor of the throughput. The relationship between PLI and throughput and can be seen graphically in Figure 3. At Phu Kham the SAG mill becomes volume constrained when throughput reaches greater than 2500 t per hour (t/h). This is the upper limit for the SAG mill.

The existing throughput model performed well with mill feed consisting of oxide (supergene) and transitional weathered mineralisation. One possible reason for the strong predictive properties of the model was a higher sampling frequency of comminution samples in the oxide and transition zones. A period of unexpected and significantly lower throughput during the fourth quarter of 2016 which coincided with a higher proportion of feed from the hypogene and included more primary metasomatic magnetite skarns. This raised uncertainty with the existing throughput model being able to predict throughput rates in the harder hypogene ore. The PLI modelling method was investigated to see why the model did not predict the lower throughputs.

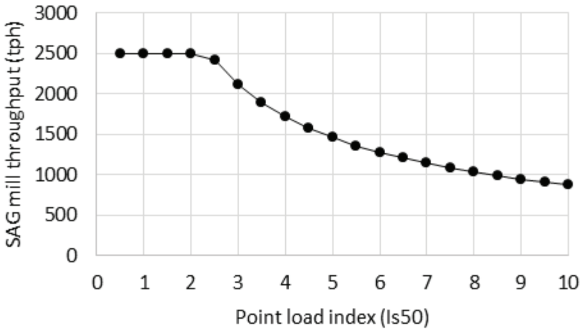

**FIG 3** – Relationship between PLI and SAG mill throughput.

Upon investigation the PLI data was found to have a fundamental problem. According to the relevant Australian Standard for testing rocks (diamond drill core) for engineering purposes, AS 4133.0–2005, the test sample must fracture in a specified way and conform to geometric rules. When the Phu Kham PLI data was checked for conformance, an unusually large number of invalid test results were included. Including invalid test results reduced the apparent PLI values and increased throughput predictions. The proportion of invalid tests at Phu Kham was 78 per cent. This unusually high number of invalid tests was attributed to samples failing along planes of weakness including foliations in the strong schistose fabric present in the sericite altered mineralised volcaniclastics. This characteristic was also identified independently from triaxial tests of metallurgical samples. A majority of results plot in the 'unlikely' region of the Deere-Miller chart, where the ratio of modulus of elasticity to compressive strength is less than 200.

The spatial location of the samples for the remaining valid PLI results was also a problem as most were near the final pit design walls, away from mineralisation. This was logical as the majority of tests were performed to support geotechnical assessment. Further spatial checks identified the average distances between valid PLI tests in the mineralised areas to be 100 to 200 m. This distance reduced near the base of the pit as inward angled geotechnical holes converged at the pit base. Given the limitations in the quantity and distribution of PLI data, there was little point estimating using a method such as Ordinary Kriging. The PLI test results were divided into domains with the purpose of assigning PLI values to the Ore Reserve block model. The domains were based on three levels of weathering (totally oxidised, partially oxidised/transition and hypogene) and nine lithologies. The statistics and histograms of the domained PLI results indicated statistically significant differences in the declustered PLI mean results at a 90 per cent confidence interval. This PLI model has proven to work in the partially weathered mineralisation and is therefore desirable to retain the model for predicting throughput rates in oxide and transition ore as cutbacks are performed.

The static throughput model (PLI – DWi – throughput) could not easily incorporate mine and mill operational improvements for the hard ore. At the SAG mill, operational improvements included adjusting the comminution circuit to pass extra work onto the ball mills and the installation of 'hard ore' liners. At the mine, high-energy blasting using quality electronic detonators was trialled which saw the powder factors increase by 1.5 times in the mineralised areas. A new modelling method was proposed which would incorporate operational improvement with factors controlling comminution performance.

Given multiple simultaneous operational improvements the contribution of individual practices could not be reliably quantified to validate relevant modelling. However, operational improvements could be readily quantified by observing the actual mill throughput data. The proposed new modelling method aimed to use geology model (weathering, lithology) and the actual mill throughput to derive an empirical model that could predict mill throughput. An added benefit of the empirical model was that it was amenable to optimising mill feed blends using linear programming (LP).

## Optimisation by linear programming

Most variables at a mine are non-linear, continuous and their models are only as strong as the assumptions they are built on. However there are a variety of applications that can be validly modelled by LP. Examples of valid applications include: blending models, allocation models, operations planning optimisation and operations scheduling.

Blending models aim to decide what mix of ingredients best fulfils the specified output requirements. Allocation models aim to divide or allocate a valuable resource among competing needs, such as land, capital, time or fuel. Operations planning optimisation aims to help a decision maker decide what to do and where to do it. Operations scheduling aims to plan when to allocate resources to meet work that is already fixed. Examples of working LP applications include blending in coal plants (Wright, 1962), methane gas hazard reduction (Chakravorty, Forrester and Richardson, 1988), open cut short-term production scheduling (Askari-Nasab *et al*, 2011), aggregate blending for concrete production (Adiguzel *et al*, 2013) and blending for asphalt mix (Awuah-Offei and Askari-Nasab, 2009).

A basic approach to solving an LP program is to use the simplex method with the aim of achieving a basic solution. The simplex algorithm characterises a problem with an objective function, defining a feasible region within which a solution exists, then considering only the extreme points to identify the desired outcome (maximum or minimum value for the objective function). The feasible region is defined by a set of rules derived from reality. For example, at Phu Kham the SAG mill throughput cannot exceed 2500 t/h. A basic solution is one which all the desired variables are solved; in some cases, such as this one, it is necessary to fix a value of one of the variables to find a unique and practical solution.

## EMPIRICAL THROUGHPUT MODELLING AND OPTIMISATION METHOD

The empirical throughput model uses the actual mill throughput with weighted causal factors. Given the ability to quantify the causal factors (tonnes of material with certain weathering, lithology) and the actual mill throughput, it is a case of determining the weighting for each factor. The empirical throughput model yields an objective function which can then be subjected to optimisation by LP.

The empirical throughput model can be determined from the sum of causal factors multiplied by a constant (Equation 1).

$$\sum_{i=1}^{n} C_i \omega_i = T \tag{1}$$

Where $C_i$ is the causal factor, $\omega_i$ is the corresponding constant and $T$ is the throughput in tonnes per hour (tph) for $i$=1...$n$ causal factors.

Three causal factors have been identified at Phu Kham based on observations from the valid PLI data. The causal factors are the proportions of fresh diorite, fresh volcaniclastic and fresh magnetite skarn in the feed. The number of causal factors is theoretically limitless but practically limited by relevance. The three factors at Phu Kham are known to be strongly causal. Given that $C_i$ and $T$ are known from measurement it is then the case of solving the simultaneous linear equations for $\omega_i$. The equation can be expressed in matrix notation (Equation 2).

$$\boldsymbol{A.B = C} \tag{2}$$

Where $A$ is the $n \times n$ non-singular square matrix of causal factors and $C$ is the corresponding vector of achieved throughputs. A square matrix is non-singular and invertible if and only if its determinant is nonzero, allowing a quick check prior to inversion:

$$det(\boldsymbol{A}) \neq 0$$

The inverted matrix $A^{-1}$ multiplied by $A$ is non-singular if the product is equal to the identity matrix $I$:

$$\boldsymbol{A^{-1}.A = A.A^{-1} = I}$$

Equation 2 can then be rearranged to solve $B$ by multiplying both sides by the inverse matrix $A^{-1}$ (Equation 3).

$$\boldsymbol{A^{-1}.A.B = A^{-1}.C} \tag{3}$$

$$\boldsymbol{I.B = A^{-1}.C} \tag{4}$$

$$\boldsymbol{B = A^{-1}.C} \tag{5}$$

Once the constants in vector B are known they can be used to predict future mill throughput using predicted causal factors. Following this fairly straightforward modelling is an opportunity to optimise the throughput. This would involve forming questions about what certain controls should be to achieve optimum value; for example, is the maximum throughput possible and what would the feed blend be to achieve the highest throughput?

The solution can be determined by linear programming. The objective function (Equation 6) and constraints (Equation 7 to Equation 11) define the system to be maximised. Firstly, an equality constraint to ensure the sum of decision variables is equal to 1, or the mill receives 100 per cent feed (Equation 7). At Phu Kham the SAG mill is volume constrained at 2500 t/h, setting the upper limit (Equation 8). Single source milling trials of the hardest, highest density feed material (metasomatic magnetite skarns) show an empirical throughput rate of 1500 t/h, setting the lower limit (Equation 9). Given that the diorite is hard and may be preferentially left out of the blend a final constraint of at least some diorite must be fed (Equation 10). Finally in order for a unique solution it is necessary to specify the variable for the amount of skarn in the feed, which in this case is a proportion of 0.1 (Equation 11). The proportion of 0.1 is based upon averages in the past dry season mining in the base of the pit.

$$\sum_{i=1}^{n} C_i \omega_i = T \tag{6}$$

$$\sum_{i=1}^{n} C_i = 1 \tag{7}$$

$$\sum_{i=1}^{n} C_i \omega_i \leq 2500 \tag{8}$$

$$\sum_{i=1}^{n} C_i \omega_i > 1500 \tag{9}$$

$$C_{diorite} > 0 \tag{10}$$

$$C_{skarn} = 0.1 \tag{11}$$

## RESULTS

### SAG Mill throughput model

The period of investigation was between May and October 2016. The tonnes of ore milled constitutes the base data from which the objective function was derived (Table 1). The objective function was then tested in November and December 2016. The difference between the predicted and actual average throughput for this period was less than 1.5 per cent relative or 30 t/h actual.

The empirical throughput model was solved using the matrix functions in Microsoft Excel. The tonnages for the fresh diorite, volcaniclastic and skarn ore were determined using 3D lithology and weathering models, end-of-month survey surfaces and the Ore Reserve model for ore/waste classification. The tonnages were converted to proportions of the mill feed. The proportion of fresh versus transitional oxide feed was checked to ensure the feed was greater than 90 per cent. Matrices $A$ and $C$ were formed. Matrix $A$ was checked to ensure its determinant was nonzero. The weightings $\omega_i$ were then derived using the previous three months data (three unknowns require three simultaneous equations). The next month's throughput was then calculated and once the actual throughput is communicated it is compared.

**TABLE 1**

Study data set.

| Month | Fresh diorite (kt) | Fresh volcaniclastics (kt) | Fresh skarn (kt) | Actual SAG mill throughput (t/h) |
|---|---|---|---|---|
| 2016–03 | 150.6 | 1128.8 | 103.3 | 2385 |
| 2016–04 | 195.9 | 1197.4 | 106.1 | 2411 |
| 2016–05 | 212.3 | 1501.8 | 205.4 | 2411 |
| 2016–06 | 583.2 | 1412.7 | 152.3 | 2395 |
| 2016–07 | 549.9 | 1272.0 | 102.0 | 2318 |
| 2016–08 | 478.8 | 1013.0 | 207.6 | 2314 |
| 2016–09 | 446.3 | 698.6 | 172.8 | 1996 |
| 2016–10 | 410.0 | 656.3 | 133.5 | 1964 |

The November and December months were used to check the predictive properties of the throughput modelling. The results can be seen in Figure 4 and Table 2. On average the throughput prediction for the two months was 2298 t/h and the actual was 2296 t/h.

### Optimised feed blend

The ideal mill blend was determined by linear programming using the simplex algorithm. A free online calculator was used (www.wolframalpha.com, 2017).

The weightings data from August, September and October 2016 data were used, which were -1869 for diorite ($x$), 3835 for volcaniclastics ($y$) and 4540 for skarn ($z$). To find the basic solution the proportion of skarn feed was set at 0.1, which is the average feed over most of 2016. The global maximum was solved by LP (Equation 12).

$$\begin{aligned} &max \left\{ \begin{matrix} -1869x + 3835y + 4540z \mid x + y + z = 1\Lambda - 1869x + 3835y + 4540z \\ \leq 2500\Lambda - 1869x + 3835y + 4540z > 1500\Lambda x > 0\Lambda z = 0.1 \end{matrix} \right\} \\ &= 2500tph \ at \ (x, y, z) \approx (0.246, 0.654, 0.1) \end{aligned} \tag{12}$$

The maximum throughput of 2500 t/h can be achieved using a blend of ~25 per cent diorite and 65 per cent volcaniclastic when the skarn feed is 10 per cent.

## CONCLUSIONS

The aim of the study was to identify a practical solution to predict SAG mill throughput using available data. The available data was the lithology/weathering geology models and the actual throughputs achieved by the SAG mill. The empirical modelling method relies on knowing the weathering and lithologies in order to make predictions. The lithology and weathering are constantly being updated from grade control drilling and pit wall and floor mapping. The geology models are considered well informed and reliable.

The empirical throughput modelling method only works as a good predictor for future mill throughputs because of good geology work and reliable 3D models. Without relevant and acceptably accurate lithology and weathering solids and surfaces the method would fail. This highlights the importance for mine geologists to constantly update and maintain relevant geology models. The Phu Kham deposit has complex geology but it is evident that the current geology modelling has adequately delineated areas of geological similarity.

The described empirical modelling method cannot be applied universally as the comminution performance of different mines are not necessarily dependent on the lithology and weathering. The method only works when the comminution controlling factors are known and have been modelled with adequate accuracy.

Repeating the method on a regular basis can yield trends in throughput performances that are a result of the sum and interaction of controlling factors. Anomalous departures from expected throughputs can be detected and investigated. The amount of work to calculate the model is low and there is no special software required.

**TABLE 2**

Empirical throughput model results used to predict November and December 2016.

| Month | Fresh diorite (kt) | Fresh volcaniclastics (kt) | Fresh skarn (kt) | Predicted SAG mill throughput (t/h) | Actual SAG mill throughput (t/h) |
|---|---|---|---|---|---|
| 2016–11 | 396.9 | 1037.6 | 99.5 | 2405 | 2371 |
| 2016–12 | 473.3 | 977.4 | 145.5 | 2190 | 2220 |

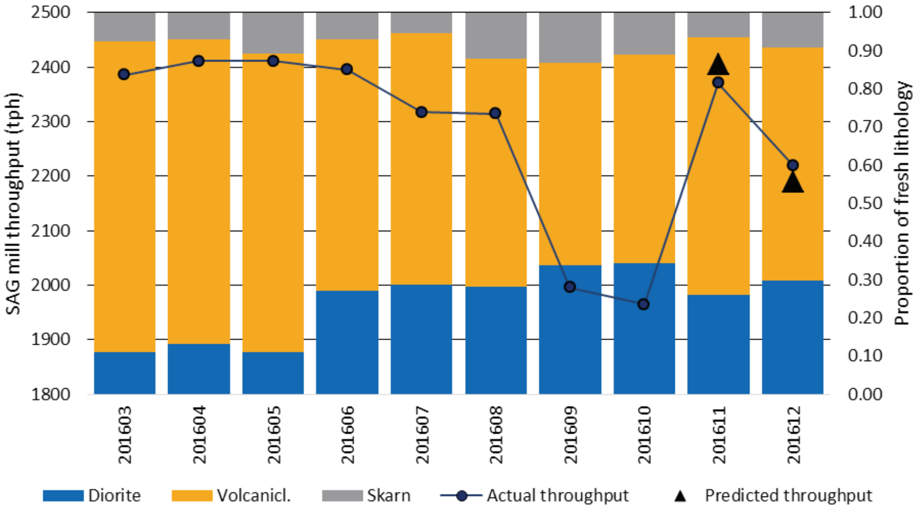

**FIG 4** – Relationship between fresh mill feed lithology proportions, actual SAG mill throughput and predicted throughput from empirical modelling.

The empirical throughput model and associated optimisation can provide timely feedback to the mine management team. The information can provide an early warning for periods of throughput that may be lower or higher than expected. Once low throughput periods are identified, mine management teams can enact mitigation measures such as adjusting LOM plans or justifying capital expenditure on items to improve throughputs.

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## REFERENCES

**Adiguzel,** D, Bascetin, A, Baray, S A and Tuylu, S, 2013. Determination of optimal aggregate blending with linear programming in concrete production, *Canadian Institute of Mining, Metallurgy and Petroleum: Bulletin*. Available from: <http://www.onemine.org/document/document.cfm?docid=214341> [Accessed: 14 March 2017].

**Askari-Nasab,** H, Eivazy, H, Tabesh, M and Badiozamani, M, 2011. A mathematical programming model for open pit short-term production scheduling, in *Proceedings SME Annual Meeting Feb 27 – Mar 02, 2011, Denver, CO*. Available from: <http://www.onemine.org/document/document.cfm?docid=201386> [Accessed: 14 March 2017].

**Awuah-Offei,** K and Askari-Nasab, H, 2009. Linear programming optimisation of asphalt mix design, in *Proceedings SME Annual Meeting Feb 22 – Feb 25, 2009, Denver, CO*. Available from: <http://www.onemine.org/document/document.cfm?docid=148879> [Accessed: 14 March 2017].

**Bennett,** D, Tordoir, A, Walker, P, La Rosa, D, Valery, W and Duffy, K, 2014. Throughput forecasting and optimisation at the Phu Kham copper-gold operation, in *Proceedings 12th AusIMM Mill Operators Conference, Townsville, Qld*, pp 381–390 (The Australian Institute of Mining and Metallurgy: Melbourne).

**Chakravorty,** R N, Forrester, P I and Richardson, W, 1988. A new technique to combat methane gas hazards, in *Proceedings Fourth International Mine Ventilation Congress, Brisbane, Queensland, July 1988*. Available from: <http://www.onemine.org/document/document.cfm?docid=195992> [Accessed: 14 March 2017].

**Kamvong,** T, Khin Zaw, Meffre, S, Maas, R, Stein, H and Chun-Kit Lai, 2014. Adakites in the Truong Son and Loei fold belts, Thailand and Laos: Genesis and implications for geodynamics and metallogeny, *Gondwana Research*, 26:165–184.

**Standards Australia,** 2005. AS 4133.0–2005 – Methods of testing rocks for engineering purposes general requirements and list of methods, December 2005.

**Tate,** N, 2005. Discovery, geology and mineralisation of the Phu Kham copper-gold deposit Lao People's Democratic Republic, in *Proceedings of the Eighth Biennial SGA Meeting* (eds: J Mao and F P Bierlein), 2:1077–1080.

**Wright,** F, 1962. Maximising the profit of a coal preparation plant by linear programming, *AIME Transactions (Mining)*, 1962, 223:82–92. Available from: <http://www.onemine.org/document/document.cfm?docid=24110> [Accessed: 14 March 2017].